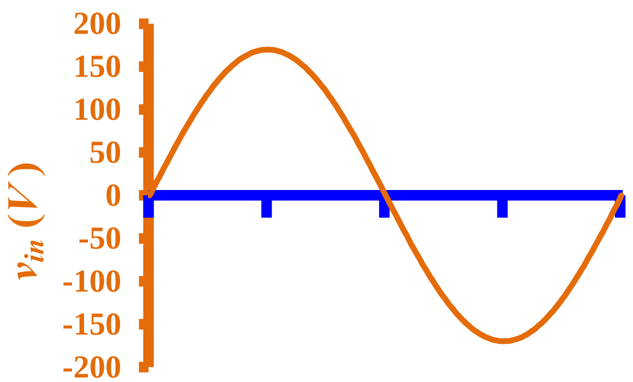
ECE 333 Green Electric Energy

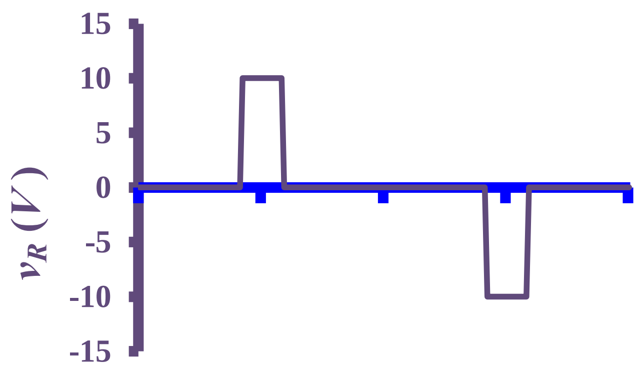
Homework 1 - Solution

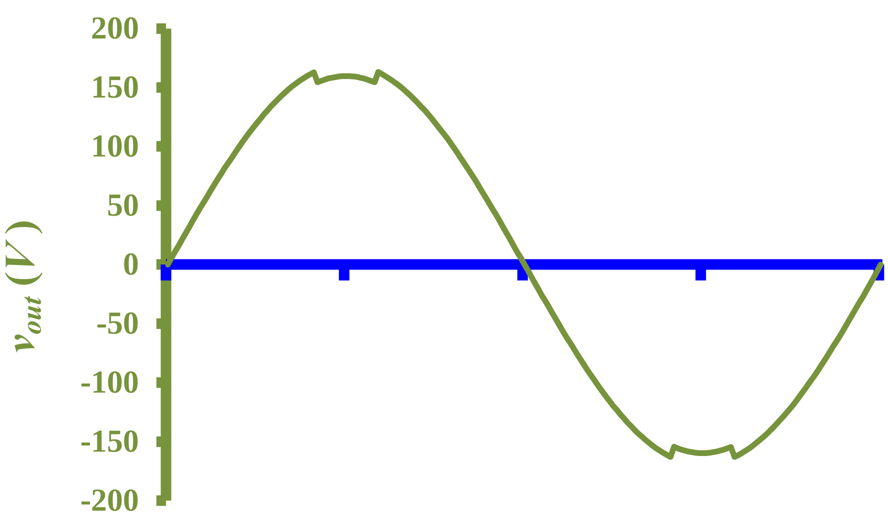
Problem 1:

$$I_{rms} = \left[\frac{1}{T}\int_0^T i^2(t)dt\right]^{\frac{1}{2}} = \left\{\frac{\int_0^2 (\frac{I}{2}t)^2 dt + \int_2^4 (I)^2 dt + \int_4^5 [I(5-t)]^2 dt}{5}\right\}^{\frac{1}{2}} = \sqrt{0.6}I\ A$$

P179-3.3:







P180-3.6:

a. reactance for the capacitor:

$$\frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 60 \times 10^{-6}} \approx 2654\Omega$$

reactance for the inductor:

$$\omega L = 2\pi f L = 2\pi \times 60 \times 7.036 \approx 2651\Omega$$

b. current through the resistor:

$$i_R(t) = \frac{v_R(t)}{R} = \frac{120\sqrt{2}\, cos\omega t}{1} = 120\sqrt{2}\, cos\omega t \ A$$

the *r.m.s.* current through the resistor:

$$I_{R,rms} = 120 \ A$$

current through the capacitor:

$$i_C(t) = \frac{120\sqrt{2}}{\frac{1}{\omega C}} cos(\omega t + \frac{\pi}{2}) \approx 0.064 cos(\omega t + \frac{\pi}{2}) \ A$$

the *r.m.s.* current through the capacitor:

$$I_{C,rms} = 0.045 \ A$$

current through the inductor:

$$i_L(t) = \frac{120\sqrt{2}}{\omega L} cos(\omega t - \frac{\pi}{2}) \approx 0.064 cos(\omega t - \frac{\pi}{2}) \ A$$

the *r.m.s.* current through the inductor:

$$I_{L,rms} = 0.045 \ A$$

c. impedance of resistor:

$$\bar{Z}_R = 1\angle 0 \ \Omega = 1 + j0 \ \Omega$$

impedance of capacitor:

$$\bar{Z}_C = \frac{1}{j\omega C} = 2654\angle -\frac{\pi}{2} \ \Omega = 0 - j2654 \ \Omega$$

impedance of inductor:

$$\bar{Z}_L = j\omega L = 2651\angle\frac{\pi}{2}\ \Omega = 0 + j2651\ \Omega$$

d. current through the resistor:

$$\bar{I}_R = 120\angle 0\ \ A$$
$$\bar{I}_R = 120 + j0\ \ A$$

current through the capacitor:

$$\bar{I}_C = 0.045\angle\frac{\pi}{2}\ \ A$$
$$\bar{I}_C = 0 + j0.045\ \ A$$

current through the inductor:

$$\bar{I}_L = 0.045\angle -\frac{\pi}{2}\ \ A$$
$$\bar{I}_L = 0 - j0.045\ \ A$$

e. total current delivered by the source:

$$\bar{I}_{Tot} = \bar{I}_R + \bar{I}_C + \bar{I}_L \approx 120\angle 0\ \ A$$

$$I_{Tot,rms} \approx 120A$$

f. power factor

$$pf \approx cos(0) = 1$$

g. total current delivered by the source:

$$i_{Tot}(t) = i_R(t) + i_C(t) + i_L(t) \approx 120\sqrt{2}cos\ \omega t\ A$$

P181-3.10:

a. real power delivered to the load

$$P = 900 \times 0.70 = 630\ kW$$

b. maximum amount of additional real power that can be delivered to the load if power factor remains unchanged:

$$\Delta P_{max} = 1000 \times 0.70 - 630 = 70\ kW$$

c. maximum amount of additional real power that can be delivered to the load if power factor is corrected to 1.0

$$\Delta P'_{max} = 1000 \times 1.0 - 630 = 370\ kW$$

P182-3.12:

Before the power factor changes:

real power delivered to the load:

$$P_1 = 750 \times 0.8 = 600 kW$$

monthly energy delivered to the load:

$$E_1 = 600 \times 720 = 432{,}000 kWh$$

Monthly bill:

$$432000 \times 0.08 + 1000 \times 10 = 44{,}560\ \$$$

After the power factor changes:

The real power remains unchanged, thus monthly energy delivered to the load remains unchanged, too. Because the real power during peak periods also remains the same, the apparent power during peak periods reduces to:

$$S_2 = \frac{1000 \times 0.8}{1} = 800 kW$$

New monthly bill:

$$432000 \times 0.08 + 800 \times 10 = 42{,}560\ \$$$

Thus 2,000 dollars could be saved each month

P184-3.15:

a. False, since the average value of the waveform is nonzero, a_0 is not zero

b. True, the function has symmetry about t = 0, *i.e.*, an even function f (t) = f (−t), this implies that it only has cosines.

c. False, see part b.

d. (skip d) False, see equation (3.101), f (t + T/2) ≠ −f (t) , there are even harmonics in the series.

P184-3.16:

a. we first determine that $\frac{2\pi}{\omega}$ is one of the periods of the current, thus:

$$I_{a,rms} = \left[\frac{1}{T}\int_0^T i_a^2(t)dt\right]^{\frac{1}{2}} = \left\{\frac{\int_0^{2\pi/\omega}(5\sqrt{2}cos(\omega t) + 4\sqrt{2}cos(3\omega t))^2 dt}{2\pi/\omega}\right\}^{\frac{1}{2}} = 6.403\,A$$

or we can compute this value directly as below:

$$I_{a,rms} = (5^2 + 4^2)^{\frac{1}{2}} = 6.403\,A$$

in the similar way, we can get

$$I_{b,rms} = I_{c,rms} = 6.403\,A$$

c. the neutral line current

$$i_n(t) = i_a(t) + i_b(t) + i_c(t) = 12\sqrt{2}cos(3\omega t)\,A$$

d. the *r.m.s* value of the neutral line current

$$I_{n,rms} = 12A$$